IL NUOVO CIMENTO Vol. 34 C, N. 4 Luglio-Agosto 2011
DOI 10.1393/ncc/i2011-10941-7

Colloquia: Channeling 2010

Coherent Schwinger scattering of fast neutrons *versus* coherent elastic nuclear scattering in a crystal

Yu. P. Kunashenko(*)

Department of Theoretical and Experimental Physics, National Research Tomsk Polytechnic University - Lenin Ave. 30, 634050 Tomsk, Russia
Department of Theoretical Physics, Tomsk State Pedagogical University - Kievskaya Str. 60 634041 Tomsk, Russia

(ricevuto il 22 Dicembre 2010; pubblicato online il 25 Agosto 2011)

Summary. — For the first time it is shown that coherent effects in particle scattering in an aligned crystal can appear due to nuclear interaction. The coherent effects at both Schwinger and coherent elastic nuclear scattering have been studied as well as the interference of fast neutrons in aligned thin crystals.

PACS 61.05.fd – Theories of neutron diffraction and scattering.
PACS 14.20.Dh – Protons and neutrons.

1. – Introduction

Neutron has an electric charge equal to zero, but Schwinger [1] predicted that fast neutrons, due to the spin (and magnetic moment) existence, can be anyhow scattered by the atomic electric field. The physics of this scattering is explained in the following way: in a neutron rest frame the magnetic field appears and the neutron magnetic moment interacts with it. Electromagnetic Schwinger scattering of fast neutrons was experimentally discovered in 1956 [2]. In the paper [3] dedicated to the semi-centenary anniversary of the discovery of Schwinger scattering for fast neutrons, a review of theoretical and experimental works on this subject, as well as the description of possible new experiments, have been done.

It is well known that when the fast charged particle penetrates into an aligned crystal, we deal with coherent scattering [4] (a thin crystal) and channeling effect (thicker crystal), which can be presented as a classical or quantum motion of particles in the electric field of continuous potential of crystallographic planes or axes [5]. In analogy with scattering of charged particles in a crystal, we may suppose that when the fast neutrons enter the crystal, one can expect both the coherent scattering of neutrons and neutron channeling.

(*) E-mail: kunashenko@tpu.ru

For the first time the coherent Schwinger scattering of neutrons in a crystal was theoretically considered in [6]. More accurate calculations of neutron coherent Schwinger scattering have been done in [7]. The results of refs. [6, 7] qualitatively show that the coherent effects for the fast neutron Schwinger scattering really take place. The detailed theory of the coherent Schwinger scattering of neutrons in a crystal was developed in [8,9].

Until now, only one experiment on fast neutron Schwinger coherent scattering in a crystal has been carried out [10]. Unfortunately, the results of this experiment do not show agreement with theoretical results of refs. [6, 7] and, strictly speaking, can be considered only as the hint on the existence of small coherent effects in Schwinger scattering of fast neutrons in a crystal. It should be underlined that the crystal thickness in the experiment [6] was 56 mm, which is too thick for investigation of coherent scattering. As shown in [9] the other reason of small coherent peak in neutron Schwinger scattering in a crystal, given in [10], might be due to the angular divergence of a neutron beam.

Channeling of fast neutrons in a crystal, due to interaction of neutron magnetic moment with continuous electrostatic potential of crystallographic planes, was shortly considered in [11, 12].

2. – Scattering of fast neutrons by a single atom

The amplitude of neutron scattering on single atom can be written as a sum of nuclear amplitude and electromagnetic amplitude [1]:

$$A = A_{\mathrm{EM}} + A_{\mathrm{NUC}}. \tag{1}$$

Therefore, the cross-section of neutron scattering by a single atom is defined by the following

$$\sigma = |A_{\mathrm{EM}}|^2 + |A_{\mathrm{NUC}}|^2 + 2A_{\mathrm{EM}} \cdot \mathrm{Im}(A_{\mathrm{NUC}})\big(\vec{P}_0 \cdot \vec{n}\big) = \sigma_{\mathrm{EM}} + \sigma_{\mathrm{NUC}} + \sigma_{\mathrm{INT}}. \tag{2}$$

Here $\vec{P}_0$ is the polarization vector of the initial neutron beam and $\vec{n}$ is the unit vector in the direction $[\vec{p}\,\vec{p}\,']$, here $\vec{p}\,(\vec{p}\,')$ is the initial (final) momentum of a neutron; σ_{EM} is the electromagnetic Schwinger neutron scattering cross-section on a single atom, σ_{NUC} is the nuclear neutron scattering cross-section and σ_{INT} is the neutron scattering cross-section due to interference of nuclear and electromagnetic interactions (interferential neutron scattering).

If the atomic screened potential is chosen in the form

$$V(r) = \frac{Ze}{r}\, e^{-r/R}, \tag{3}$$

where R is the screening radius, the differential cross-section for Schwinger electromagnetic scattering (σ_{EM}) of neutrons by an individual atom can be written now as follows:

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}\Omega} = \frac{\gamma_n^2}{4}\left(\frac{Ze^2}{M}\right)^2 \frac{q^4}{(q^2 + R^{-2})^2}\,\mathrm{ctg}^2\frac{\theta}{2}\,. \tag{4}$$

Here, q is the transferred momentum, Z is the atomic number, $F(q)$ is the atomic form factor, M is the neutron mass, $\gamma_n = 1.91$ is the anomalous magnetic moment of

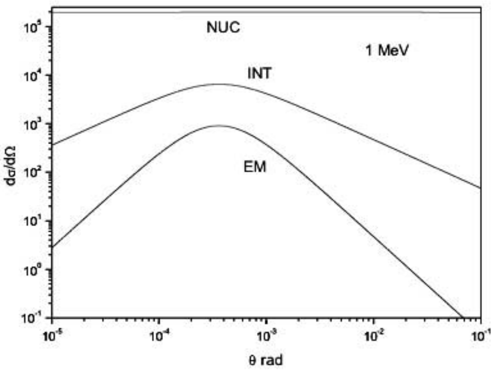

Fig. 1. – The neutron electromagnetic scattering cross-section on the individual tungsten atom (EM), the nuclear neutron scattering cross-section (NUC) and the interferential neutron scattering cross-section (INT) for the case $\vec{P}_0\vec{n} = 1$ as a function of the neutron scattering angle. Other parameters: the target atom is W, the neutron kinetic energy is equal to 1 MeV.

a neutron in nuclear magnetons, θ is the neutron scattering angle, $\vec{q} = \vec{p} - \vec{p}\,'$ is the transferred to atom momentum with([1])

$$q^2 = 2p^2(1 - \cos\theta) = 4p^2 \sin^2 \frac{\theta}{2}\,.$$

In order to estimate the nuclear amplitude of neutron scattering on a single atom we have used a simple three-dimensional rectangular potential well, with the well depth $U + iW$ and the width R_N, we have chosen $U = -52.5 + 0.6\,E$, $W = -2.5 + 0.3\,E$, where E is the neutron energy and $R_N = 1.37\ A^{1/3}\,\mathrm{Fm}$, where A is the mass number. The nuclear neutron scattering cross-section is

$$\frac{\mathrm{d}\sigma_{\mathrm{NUC}}}{\mathrm{d}\Omega} = 4R_N^2(MR_N)^2(U^2 + W^2)\frac{(-qR_N\cos(qR_N) + \sin(qR_N))^2}{(qR_N)^6}\,, \tag{5}$$

and the interferential neutron scattering cross-section is

$$\frac{\mathrm{d}\sigma_{\mathrm{INT}}}{\mathrm{d}\Omega} = 2MR_N^2\, W\gamma_n e^2 \frac{q^2}{q^2 + R^{-2}} \frac{-qR_N\cos(qR_N) + \sin(qR_N)}{(qR_N)^3} \cot\frac{\theta}{2}(\vec{P}_0 \cdot \vec{n}). \tag{6}$$

Figure 1 shows the angular dependence of electromagnetic Schwinger neutron scattering cross-section on an individual tungsten atom (EM), the angular dependence of the nuclear neutron scattering cross-section (NUC) and the angular dependence of the interferential neutron scattering cross-section (INT) for the case when the initial neutron beam is completely polarized and $(\vec{P}_0 \cdot \vec{n}) = 1$. The kinetic energy of neutrons has been chosen to be equal to 1 MeV.

([1]) The units $\hbar = c = 1$ are used.

3. – Scattering of fast neutrons in a crystal

It is well known that in an aligned crystal the scattering cross-section should be written as a sum of coherent $\mathrm{d}\sigma_{\mathrm{coh}}$ and incoherent $\mathrm{d}\sigma_{\mathrm{incoh}}$ parts [4, 13]:

$$\mathrm{d}\sigma_{\mathrm{cr}} = \mathrm{d}\sigma_{\mathrm{coh}} + \mathrm{d}\sigma_{\mathrm{incoh}}; \tag{7}$$

$$\mathrm{d}\sigma_{\mathrm{coh}} = I(\vec{q})|S(\vec{q})|^2 \exp\left[-q^2\overline{u^2}\right] \mathrm{d}\sigma_i; \tag{8}$$

$$\mathrm{d}\sigma_{\mathrm{incoh}} = N_{\mathrm{tot}}\left[1 - \exp\left[-q^2\overline{u^2}\right]\right] \mathrm{d}\sigma_i. \tag{9}$$

Here $\mathrm{d}\sigma_i$ is one of the possible scattering neutron cross-sections on the individual atom of a crystal: σ_{EM} is the electromagnetic Schwinger neutron scattering cross-section on the single atom, σ_{NUC} is the nuclear neutron scattering cross-section and σ_{INT} is the interferential neutron scattering; $I(\vec{q})$ is the interferential multiplier responsible for the appearance of coherent effects, the exponential $\exp[-q^2\overline{u^2}]$ is the Debye-Waller factor, which takes into account the thermal vibrations of crystal atoms, $\overline{u^2}$ the mean-square displacement of crystal atoms from equilibrium positions, $N_{\mathrm{tot}} = N_X N_Y N_Z$ is the total number of atoms in a crystal, $S(\vec{q})$ is the crystal structure factor. In a reference system connected with the crystal the interferential multiplier has the form [13]

$$I(\vec{q}\,') = N_X N_Y \frac{(2\pi)^2}{d_y d_x} \sum_{n,l} \delta(q'_x - g_x n)\delta(q'_y - g_y m) \frac{\sin^2\left(\frac{1}{2}N_Z q'_z d_z\right)}{\sin^2\left(\frac{1}{2}q'_z d_z\right)}, \tag{10}$$

where $\vec{q}\,'$ is the transferred momentum in the crystal coordinate system, N_X, N_Y and N_Z are the numbers of crystal atoms in X, Y and Z directions, respectively, which contribute to the coherent process, $g_x = 2\pi/a_x$, $g_y = 2\pi/a_y$ are one-dimensional reciprocal lattice vectors in X, Y directions, respectively, a_x, a_y are the lattice constants. Expression (9) for the interferential multiplier is valid when $N_X \gg 1$, $N_Y \gg 1$ and $N_Z \ll \{N_X, N_Y\}$.

To proceed further with calculations, it is convenient to rewrite the cross-sections of neutron scattering in terms of transferred momentum. If the OZ axis is directed along the initial neutron momentum, then the X and Y components of the transferred momentum are

$$\begin{aligned} q_x &= p\cos\varphi\sin\theta, \\ q_y &= p\sin\varphi\sin\theta. \end{aligned} \tag{11}$$

Here p is the initial neutron momentum, θ and ϕ are the neutron scattering angles in the 2nd coordinate system. Expressing the angles θ and ϕ in terms of the transferred momentum one finds

$$\theta(q_x, q_y) = \arcsin\left(\frac{\sqrt{q_x^2 + q_y^2}}{p}\right), \qquad \varphi(q_x, q_y) = \arccos\left(\frac{q_x}{\sqrt{q_x^2 + q_y^2}}\right), \tag{12}$$

and for $\mathrm{d}\Omega$

$$\mathrm{d}\Omega = \frac{\mathrm{d}q_y \mathrm{d}q_x}{p\sqrt{p^2 - q_x^2 - q_y^2}}. \tag{13}$$

Finally, the expression for the differential scattering cross-section of Schwinger scattering can be presented as

$$\frac{\mathrm{d}\sigma_{\mathrm{EM}}(\vec{q})}{\mathrm{d}q_x\mathrm{d}q_y} = \frac{\gamma_n^2}{4}\left(\frac{Ze^2}{M}\right)^2 \frac{q^4}{(q^2+R^{-2})^2} \frac{p+\sqrt{p^2-q_x^2-q_y^2}}{p-\sqrt{p^2-q_x^2-q_y^2}} \frac{1}{p\sqrt{p^2-q_x^2-q_y^2}}, \tag{14}$$

the nuclear neutron scattering cross-section as

$$\frac{\mathrm{d}\sigma_{\mathrm{NUC}}(\vec{q})}{\mathrm{d}q_x\mathrm{d}q_y} = 4R_N^2(MR_N)^2(U^2+W^2)\frac{(-qR_N\cos(qR_N)+\sin(qR_N))^2}{(qR_N)^6} \times \\ \times\frac{1}{p\sqrt{p^2-q_x^2-q_y^2}}, \tag{15}$$

and the interferential neutron scattering cross-section as

$$\frac{\mathrm{d}\sigma_{\mathrm{INT}}(\vec{q})}{\mathrm{d}q_x\mathrm{d}q_y} = 2MR_N^2\, W\gamma_n e^2\frac{q^2}{(q^2+R^{-2})}\frac{-qR_N\cos(qR_N)+\sin(qR_N)}{(qR_N)^3} \times \\ \times\frac{p+\sqrt{p^2-q_x^2-q_y^2}}{p-\sqrt{p^2-q_x^2-q_y^2}}\frac{(\vec{P}_0\cdot\vec{n}\,)}{p\sqrt{p^2-q_x^2-q_y^2}}. \tag{16}$$

Formulas (10)–(16) are written in a coordinate system connected with the neutron, but the formula for the interferential multiplier is written in a coordinate system connected with the crystal.

The transformation of the transferred momentum from one coordinate system to another is given by the following expressions [4]:

$$\begin{cases} q'_x = q_z\sin\Psi - (q_x\sin\Phi - q_y\cos\Phi)\cos\Psi, \\ q'_y = q_x\cos\Phi + q_y\sin\Phi, \\ q'_z = q_z\cos\Psi - (q_x\sin\Phi - q_y\cos\Phi)\sin\Psi. \end{cases} \tag{17}$$

with Ψ being the angle between $\vec{p}$ and the OZ axis and Φ the angle between the $\vec{p}$ projection onto the XOY plane and the OX axis. The neutron initial angles are shown in fig. 2.

In a real experiment, one always detects the neutron scattered within definite range of the angles. For example, if one measures the neutron scattered within the solid angle $\vartheta = 0$–$\Theta_{\max}$, $\varphi = 0$–2π, then we have to integrate the Schwinger differential scattering cross-section over the scattering angles

$$\sigma_{\mathrm{cr}} = \int_{\Delta\Omega}\frac{\mathrm{d}\sigma_{\mathrm{cr}}}{\mathrm{d}\Omega}\mathrm{d}\Omega = \sigma_{\mathrm{coh}}+\sigma_{\mathrm{incoh}} = \int_{\Delta\Omega}\frac{\mathrm{d}\sigma_{\mathrm{coh}}}{\mathrm{d}\Omega}\mathrm{d}\Omega + \int_{\Delta\Omega}\frac{\mathrm{d}\sigma_{\mathrm{incoh}}}{\mathrm{d}\Omega}\mathrm{d}\Omega = \\ = \int_0^{2\pi}\mathrm{d}\varphi\int_0^{\Theta\max}\sin\theta\mathrm{d}\theta\frac{\mathrm{d}\sigma_{\mathrm{coh}}}{\mathrm{d}\Omega}(\theta,\varphi) + \int_0^{2\pi}\mathrm{d}\varphi\int_0^{\Theta_{\max}}\sin\theta\mathrm{d}\theta\frac{\mathrm{d}\sigma_{\mathrm{incoh}}}{\mathrm{d}\Omega}(\theta,\varphi). \tag{18}$$

Here $\Theta_{\max}$ is the maximal scattering angle of neutrons with respect to the initial momentum direction (angular size of the collimator).

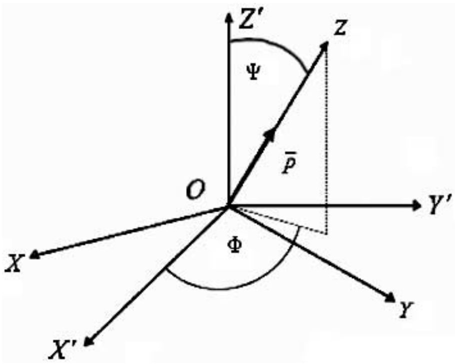

Fig. 2. – Two coordinate systems used to describe the neutron scattering in a crystal. The coordinate system XYZ is linked to the crystal: the OY axis coincides with the crystallographic axis $\langle 100\rangle$ and the plane XOZ coincides with the crystallographic plane (100). The OZ' axis of the second coordinate system $X'Y'Z'$ is linked to the neutron momentum $\vec{p}$, Ψ is the angle between $\vec{p}$ and the OZ axis and Φ the angle between the $\vec{p}$ projection onto the XOY plane and the OX axis.

After substitution of eqs. (7), (9), (13) into eq. (15) and integration over permitted angles we have derived the following formula for the coherent part of the cross-section:

$$\sigma_{\mathrm{coh}}(\Theta_{\max}) = \frac{\gamma_n^2}{4}\left(\frac{Ze^2}{M}\right)^2 N_x N_y \frac{(2\pi)^2}{d_y d_x}\sum_{n,m}\left\{\frac{\mathrm{d}\sigma_i\left(\vec{q}\,'_{\perp nm}, q'_{z\ nm}\right)}{\mathrm{d}q_x \mathrm{d}q_y}\times\right.$$
$$\left.\times\frac{\sin^2\left(\frac{1}{2}N_z q'_{z\ nm} d_z\right)}{\sin^2\left(\frac{1}{2}q'_{z\ nm} d_z\right)}\left|S\left(\vec{q}\,'_{\perp nm}, q'_{z\ nm}\right)\right|^2\right\}, \tag{19}$$

in which we introduced the following notations: $\vec{q}\,'_{\perp nm}$ is the two-dimensional vector in a plane perpendicular to the OZ axis with coordinates $(g_x^n,\, g_y^m)$ and $q'_{znm} = p - \sqrt{p^2 - {q'}^2_{\perp nm}}$ is the projection of the neutron transferred momentum onto the OZ axis.

The summation in eq. (16) should be performed taking into account the condition

$${q'}^2_{\perp nm} \le p^2 \sin^2\Theta_{\max}.$$

As it follows from eq. (16), the coherent part of the cross-section exhibits sharp peaks when

$$\frac{1}{2}q'_{znm} d_z = \pi k, \quad k = 1, 2, 3, \ldots$$

The formula for the incoherent part of the cross-section integrated over permitted scattering angles becomes

$$\sigma_{\mathrm{incoh}}(\Theta_{\max}) = N_{\mathrm{tot}}\int_{\Delta\Omega}\left[1 - \exp\left[-q^2(\theta,\varphi)\overline{u^2}\right]\right]\frac{\mathrm{d}\sigma_i}{\mathrm{d}\Omega}(\theta,\varphi)\mathrm{d}\Omega.$$

Here, $N_{\mathrm{tot}} = N_X N_Y N_Z$.

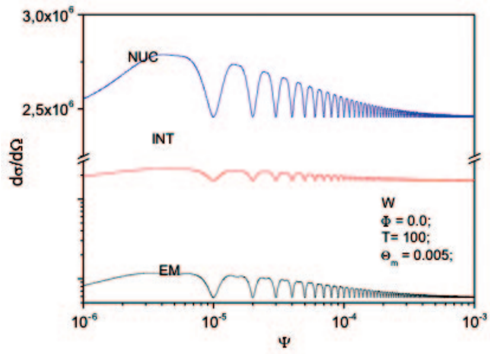

Fig. 3. – Orientation dependence of the coherent neutron scattering in W ⟨100⟩ crystal.

In fig. 3 we have presented the orientation dependence of the coherent electromagnetic Schwinger neutron scattering cross-section in W ⟨100⟩ crystal (EM); the orientation dependence of the nuclear neutron scattering cross-section (NUC); the orientation dependence of the interferential neutron scattering cross-section (INT) for the case $(\vec{P}_0 \cdot \vec{n}) = 1$ in W ⟨100⟩ crystal. The kinetic energy of the neutrons is equal to 1 MeV, $\Theta_{\max} = 5$ mrad.

4. – Conclusions

For the first time it is shown that coherent effects at particle scattering in an aligned crystal can appear due to nuclear interaction.

The results of calculations show that coherent effects exist not only for electromagnetic Schwinger scattering, but also for both nuclear and interferential scattering of fast neutrons.

The cross-sections of electromagnetic Schwinger, nuclear and interferential scattering of fast neutrons are characterized by a coherent peak at definite alignment of the crystal with respect to the incident neutron beam. This can be used to suggest a new experimental scheme to observe the Schwinger (electromagnetic) scattering of fast neutrons by atoms.

In the future we plan to consider the polarization of a fast neutron beam after passage through the crystal. The calculations with more realistic nuclear potential taking into account the spin-orbit interaction are in progress, and the results will be published elsewhere.

* * *

The author is grateful to Prof. Yu. L. Pivovarov for the useful discussions. The work is partially supported by Russian Science and Innovations Federal Agency under contract No 02.740.11.0238; by Russian Fund for Basic Research, Grant 10-02-013869; by Russian Education Federal Agency program "Support of Scientific Schools", contract 3558.2010.2.

REFERENCES

[1] Schwinger J., *Phys. Rev.*, **73** (1948) 407.
[2] Alexandrov Yu. A. and Bondarenko I. I., *Zh. Exsp. Teor. Fiz.*, **31** (1956) 726 (in Russian).

[3] Alexandrov Yu. A., *To semicentenary anniversary of discovering and starting the first works on neutron polarization*, Preprint E3-2006-142 JINR (2006).
[4] Ter-Mikelian M. L., *High Energy Electromagnetic processes in Condensed Media* (Wiley Interscience, New-York) 1972.
[5] Baier V. N., Katkov V. M. and Strakhovenko V. M., *Electromagnetic Processes at High Energies in Oriented Single Crystals* (World Scientific Publishing Co, Singapore) 1998.
[6] Dyumin A. N. *et al.*, *JETP Lett.*, **31** (1980) 413.
[7] Baryshevskii V. G. and Zaitzeva A. M., *Izv. Vyssh. Uchebn. Zaved.*, **3** (1985) 103 (in Russian).
[8] Kunashenko Yu.P. and Pivovarov Yu.L., *Charged & Neutral Particles Channeling Phenomena (Channeling 2008): Proceedings of the 51st Workshop of the INFN Eloisatron Project* (World Scientific) 2010, p. 794.
[9] Kunashenko Yu. P., to be published in *Poverkhn. Rentgenov., Sinkhrotron. Neitron. Issled.*
[10] Dyumin A. N. *et al.*, *JETP Lett.*, **42** (1985) 61.
[11] Vysotskii V. I. and Kusmin R. N., *Zh. Exsp. Teor. Fiz.*, **82** (1982) 177 (in Russian).
[12] Vysotskii V. I. and Vysotskii M. V., *Poverkhn. Rentgenov., Sinkhrotron. Neitron. Issled.*, **4** (2010) 100 (in Russian).
[13] Kunashenko Yu. P., *Nucl. Instrum. Methods*, **229** (2005) 219.